Unit 7

Quadratics

Honors Algebra 2

Quadratics & Solving Quadratic Equations by Factoring

Quadratics are polynomials that have a degree of 2.

Linear:	$2x + 5$
Quadratic:	$3x^2 - 4x + 7$
Cubic	$x^3 - 6x^2 + x - 1$

When solving quadratic equations, we will have a few options, each of which have restrictions, advantages and disadvantages.

We already have used one method for solving a quadratic!

The Zero Product Property

If $ab = 0$, then $a = 0$ or $b = 0$

METHOD 1: Solving a Quadratic Equation by Factoring

- Get one side of the equation $= 0$
- Factor the non-zero side
- Set each factor equal to zero and solve (ZPP)

Example 1: Solve.

a. $x^2 - 5x + 6 = 0$ b. $2x^2 = 8x$ c. $2x^2 + 6x = -4$

Solutions on Graphs

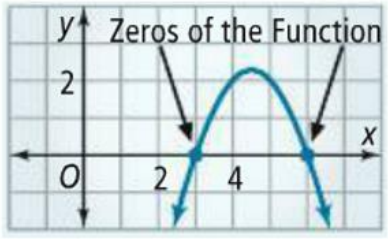
Example 2:

$$x^2 = x + 30$$
$$x^2 - x - 30 = 0$$
$$(x - 6)(x + 5) = 0$$

$x - 6 = 0$ $\qquad$ $x + 5 = 0$

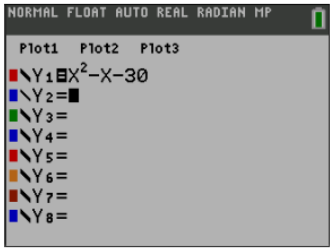
$x = 6$ $\qquad$ $x = -5$

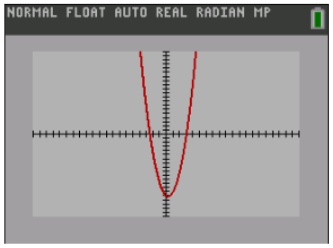
We can also use graphing calculators:

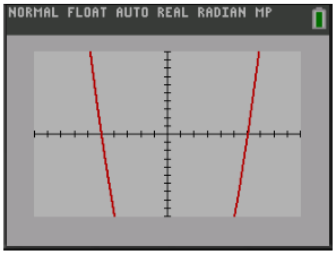


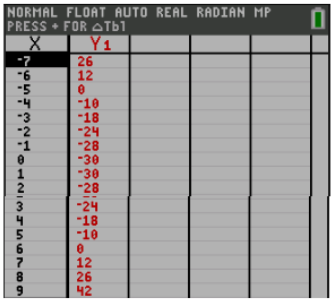
The zeros of a function are the x-coordinates where the graph crosses the x-axis.

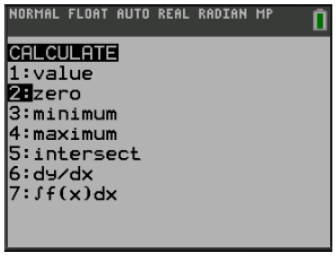
(They are the points where the y-coordinate is 0.)

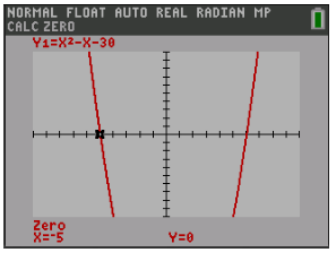


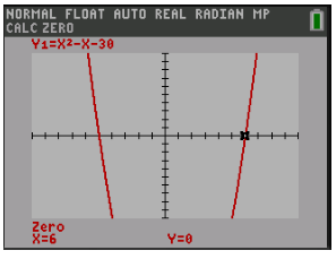












Homework #19

Part 1: Solve by factoring.

1.) $x^2 - 2x - 24 = 0$

2.) $3x^2 = x + 4$

3.) $4x^2 + 6x + 5 = 5$

4.) $(x + 2)^2 = 49$

* You have to expand $(x + 2)^2$ first...

Part 2: Solve using a calculator.

5.) $10x^2 = 4 - 3x$

6.) $3x^2 + 2x = 2$

7.) $4x^2 - x = 6$

8.) $5 - x = \frac{1}{2}x^2$

We can use the following techniques when there's no x term…

…or when all the xs are in parentheses (like example 3,6, or 9).

Example 1:

$x^2 = 25$

Example 2:

$x^2 = 192$

Example 3:

$(x+3)^2 = 49$

Example 4:

$x^2 - 9 = 0$

Example 5:

$x^2 - 50 = 0$

Example 6:

$(x-5)^2 - 100 = 0$

Example 7:

$x^2 - 4 = 60$

Example 8:

$10x^2 + 9 = 499$

Example 9:

$(x+12)^2 - 50 = 400$

METHOD 2: Solving a Quadratic Equation by taking a Square Root

- Get the x^2 or the $(\quad)^2$ by itself on the left of the =
- Take the square root of both sides … add $\pm$
- Solve what remains for x

Homework #20

1.) $x^2 = 121$

2.) $x^2 = 540$

3.) $x^2 + 1 = 2$

4.) $x^2 - 252 = 0$

5.) $x^2 - 186 = 300$

6.) $6x^2 = 150$

7.) $7x^2 - 6 = 57$

8.) $3x^2 + 7 = 301$

9.) $(2x - 1)^2 = 9$

10.) $(6x + 2)^2 + 4 = 28$

Completing the Square

If we could make $x^2 - 6x - 3 = 0$ look more like a problem from our last lesson, we could just take the square root!

$$(2x-1)^2 = 9$$

$$\sqrt{(2x-1)^2} = \pm\sqrt{9}$$

But not all polynomials can be factored into a $(\quad)^2$ ...

...unless there was a method to FORCE them to factor that way!

$$ax^2 + bx + c = 0$$

METHOD 3: Solving a Quadratic Equation by Completing the Square

- Isolate the constant term on the right side of the equation
- If the coefficient of the quadratic term, a, is not 1, then divide both sides by a
- Calculate and then add $\left(\frac{b}{2a}\right)^2$ to both sides of the equation
- Factor the left side as the square of a binomial
- Solve by taking the square root of both sides, don't forget the $\pm$

Example 1: $x^2 - 6x - 3 = 0$

Example 2: $2x^2 + 4x + 1 = 0$

Complex Solutions to Quadratic Equations

Example 3: $2x^2 + 2x + 5 = 0$

Quadratic Equations in Disguise

Example 4: $\frac{1}{x+1} + \frac{1}{x-1} = 1$

Example 5: $\sqrt{x+3} = 2x$

Homework #21

Part 1: Solve by completing the square.

1.) $y^2 + 6y - 2 = 0$

2.) $k^2 - 10k + 30 = 0$

3.) $t^2 + 8 = 4t$

4.) $2n^2 - 8n - 3 = 0$

5.) $2x(x - 4) = 3(1 - x)$

6.) $\frac{1}{y+2} + \frac{1}{y+6} = 1$

Completing the Square to Develop the Quadratic Formula

Solve $ax^2 + bx + c = 0$ for x in terms of a, b and c.

The Quadratic Formula

When given a quadratic of the form $ax^2 + bx + c = 0$, the solutions to the equation can be found using the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1: $3x^2 + x - 1 = 0$

Example 2: $5y^2 = 6y - 3$

Homework #22

Part 1: Solve using the quadratic formula.

1.) $x^2 - 4x + 13 = 0$

2.) $5k^2 + 3k - 2 = 0$

3.) $2p^2 - 3p - 2 = 0$

4.) $3y^2 = 1 - y$

5.) $2x(x - 1) = 7$

6.) $(2x + 1)(2x - 1) = 4x$

7.) $\frac{w^2}{2} - w = \frac{3}{4}$

8.) $\frac{4-2t^2}{7} = 2t$

The Discriminant

dis·crim·i·nate [*v.* dih-**skrim**-*uh*-neyt; *adj.* dih-**skrim**-*uh*-nit] Show IPA ***verb,* -nat·ed, -nat·ing, *adjective***

–verb (used without object)

1. to make a distinction in favor of or against a person or thing on the basis of the group, class, or category to which the person or thing belongs rather than according to actual merit; show partiality: *The new law discriminates against foreigners. He discriminates in favor of his relatives.*
2. to note or observe a difference; distinguish accurately: *to discriminate between things.*

–verb (used with object)

3. to make or constitute a distinction in or between; differentiate: *a mark that discriminates the original from the copy.*
4. to note or distinguish as different: *He can discriminate minute variations in tone.*

www.dictionary.com

The discriminant is the "stuff" inside the square root in the quadratic formula. (It does not include the square root though)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \leftarrow \text{discriminant}$$

If $b^2 - 4ac > 0$

The discriminant is positive

- > 0 and a perfect square
- > 0 and NOT a perfect square

If $b^2 - 4ac = 0$

The discriminant is zero

If $b^2 - 4ac < 0$

The discriminant is negative

Discriminant > 0 and Perfect Square (example)

Example 1: $4x^2 + 11x - 20 = 0$

Calculate the value of the discriminant:	Determine the number and type of solutions: 1 Solution or 2 Solutions Rational or Irrational or Complex	Calculate the actual answers using the quadratic formula:	Do your solutions match what you determined using the discriminant?

Discriminant > 0 and NOT a Perfect Square (example)

Example 2: $x^2 + 5 = -5x$

Calculate the value of the discriminant:	Determine the number and type of solutions: 1 Solution or 2 Solutions Rational or Irrational or Complex	Calculate the actual answers using the quadratic formula:	Do your solutions match what you determined using the discriminant?

Discriminant $= 0$ (example)

Example 3: $5x^2 + 50x = -125$

Calculate the value of the discriminant:	Determine the number and type of solutions: 1 Solution or 2 Solutions Rational or Irrational or Complex	Calculate the actual answers using the quadratic formula:	Do your solutions match what you determined using the discriminant?

Discriminant < 0 (example)

Example 4: $x^2 - 4x + 9 = 0$

Calculate the value of the discriminant:	Determine the number and type of solutions: 1 Solution or 2 Solutions Rational or Irrational or Complex	Calculate the actual answers using the quadratic formula:	Do your solutions match what you determined using the discriminant?

The Discriminant (Summarized)

If the discriminant is...	Then the solution(s) to the quadratic equation will be...
Negative	2 complex solutions
Zero	1 real (rational) solution
Positive (perfect square)	2 real (rational) solutions
Positive (in general)	2 real (irrational) solution

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \leftarrow \text{discriminant}$$

Homework #23

For each of the following…

1. **Calculate the value of the discriminant**
2. **State the number and type of solutions you will get (see examples in class for your choices)**

1. $x^2 + 6x + 3 = 0$

2. $x^2 + 6x + 5 = 0$

3. $x^2 + 8x + 16 = 0$

4. $x^2 + 6x + 10 = 0$

5. $x^2 - 5x - 5 = 0$

6. $3x^2 - 4x + 2 = 0$