

Section 2-5: Reasoning in Algebra and Geometry

Take note

Key Concept Properties of Equality

Let a , b , and c be any real numbers.

Addition Property	If $a = b$, then $a + c = b + c$.
Subtraction Property	If $a = b$, then $a - c = b - c$.
Multiplication Property	If $a = b$, then $a \cdot c = b \cdot c$.
Division Property	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Reflexive Property	$a = a$
Symmetric Property	If $a = b$, then $b = a$.
Transitive Property	If $a = b$ and $b = c$, then $a = c$.
Substitution Property	If $a = b$, then b can replace a in any expression.

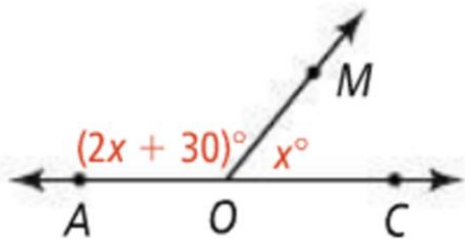
Take note

Key Concept The Distributive Property

Use multiplication to distribute a to each term of the sum or difference within the parentheses.

Sum: $a(b + c) = a(b + c) = ab + ac$

Difference: $a(b - c) = a(b - c) = ab - ac$



What is the value of x .

Justify each step.

$\angle AOM$ and $\angle MOC$ are supplementary.	\sphericalangle s that form a linear pair are supplementary.
$m\angle AOM + m\angle MOC = 180$	Definition of supplementary \sphericalangle s
$(2x + 30) + x = 180$	Substitution Property
$3x + 30 = 180$	Like terms can be combined
$3x = 150$	Subtraction Property of Equality
$x = 50$	Division Property of Equality

What is the value of x ? Justify each step.

Given: \overrightarrow{AB} bisects $\angle RAN$.



\overrightarrow{AB} bisects $\angle RAN$	Given
$m\angle RAB = m\angle BAN$	Def. of an angle bisector
$x = 2x - 75$	Substitution property
$-x = -75$	Subtraction property of equality
$x = 75$	Multiplication (or Div) property of equality

Take note

Key Concept Properties of Congruence

Reflexive Property

$$\overline{AB} \cong \overline{AB} \quad \angle A \cong \angle A$$

Symmetric Property

$$\text{If } \overline{AB} \cong \overline{CD}, \text{ then } \overline{CD} \cong \overline{AB}.$$

$$\text{If } \angle A \cong \angle B, \text{ then } \angle B \cong \angle A.$$

Transitive Property

$$\text{If } \overline{AB} \cong \overline{CD} \text{ and } \overline{CD} \cong \overline{EF}, \text{ then } \overline{AB} \cong \overline{EF}.$$

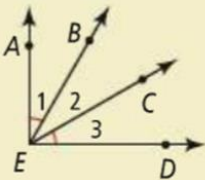
$$\text{If } \angle A \cong \angle B \text{ and } \angle B \cong \angle C, \text{ then } \angle A \cong \angle C.$$

$$\text{If } \angle B \cong \angle A \text{ and } \angle B \cong \angle C, \text{ then } \angle A \cong \angle C.$$

A proof is a convincing argument that uses deductive reasoning.

A two-column proof lists each new conclusion on the left and the reason why you make that new conclusion on the right.

Given: $m\angle 1 = m\angle 3$
Prove: $m\angle AEC = m\angle DEB$



Statements	Reasons
1) $m\angle 1 = m\angle 3$	1) Given
2) ~~~~~	2) ~~~~~
3) ~~~~~	3) ~~~~~
4) ~~~~~	4) ~~~~~
5) $m\angle AEC = m\angle DEB$	5) ~~~~~

The first statement is usually the given statement.

Each statement should follow logically from the previous statements.

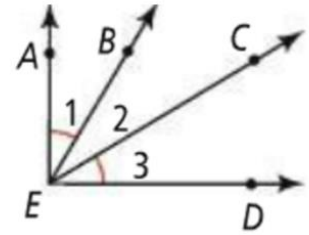
The last statement is what you want to prove.

*When you write these, you will have to control your urge to just get the answer. You must justify each step - even steps you would otherwise skip!

Write a two-column proof.

Given: $m\angle 1 = m\angle 3$

Prove: $m\angle AEC = m\angle DEB$



Statements	Reasons
1) $m\angle 1 = m\angle 3$	1) Given
2) $m\angle 2 = m\angle 2$	2) Reflexive Property of Equality
3) $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	3) Addition Property of Equality
4) $m\angle 1 + m\angle 2 = m\angle AEC$ $m\angle 3 + m\angle 2 = m\angle DEB$	4) Angle Addition Postulate
5) $m\angle AEC = m\angle DEB$	5) Substitution Property

a. Write a two-column proof.

Given: $\overline{AB} \cong \overline{CD}$

Prove: $\overline{AC} \cong \overline{BD}$



Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$	1. Given
2. $AB = CD$	2. Congruent segments have equal lengths
3. $BC = BC$	3. Reflexive Property
4. $AB + BC = CD + BC$	4. Addition Property
5. $AB + BC = AC$	5. Segment addition postulate
6. $CD + BC = BD$	6. Segment addition postulate
7. $AC = BD$	7. substitution
8. $\overline{AC} \cong \overline{BD}$	8. Segments with = length are \cong