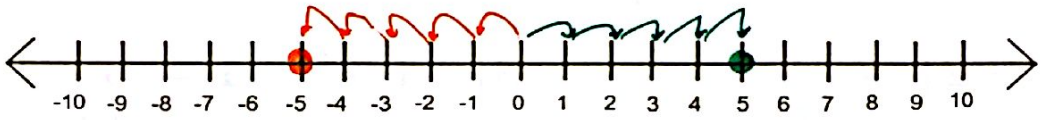
Section 3-7: Absolute Value Equations and Inequalities

Absolute Value is the distance from the origin.

$$|x| = 5$$



-5 or 5

Example 1: Solve the absolute value equations

a. $|x| - 5 = -2$

$+5 \quad +5$

$|x| = 3$

$x = 3$ or $x = -3$

b. $3|x| + 2 = 14$

$-2 \quad -2$

$\frac{3|x|}{3} = \frac{12}{3}$

$|x| = 4$

$x = 4$ or $x = -4$

Example 2: Solve the absolute value equations

a. $|3x| = 18$

$\frac{3x}{3} = \frac{18}{3}$ or $\frac{3x}{3} = \frac{-18}{3}$

$x = 6$ or $x = -6$

b. $|x + 7| = 3$

$x + 7 = 3$ or $x + 7 = -3$

$-7 \quad -7 \qquad -7 \quad -7$

$x = -4$ or $x = -10$

- Isolate the absolute value FIRST
- Separate into two equations
  - On the left - just remove the absolute value
  - On the right - remove absolute value and make the right side negative
- Solve each separately

Example 3: Solve the absolute value equations

a. $\frac{2|3x-2|}{2} = \frac{14}{2}$

$|3x-2| = 7$

$3x-2=7$ or $3x-2=-7$

$+2 \quad +2 \qquad +2 \quad +2$

$\frac{3x}{3} = \frac{9}{3} \qquad \frac{3x}{3} = \frac{-5}{3}$

$x=3$ or $x=\frac{-5}{3}$

b. $3|x+2|-1=8$

$+1 \quad +1$

$\frac{3|x+2|}{3} = \frac{9}{3}$

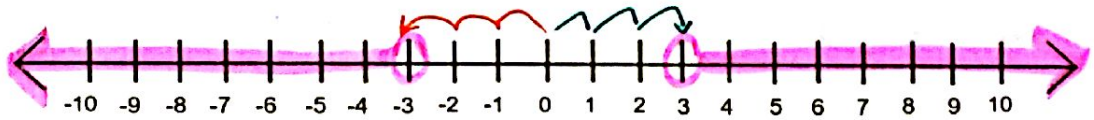
$|x+2| = 3$

$x+2=3$ or $x+2=-3$

$-2 \quad -2 \qquad -2 \quad -2$

$x=1$ or $x=-5$

$|x| > 3$

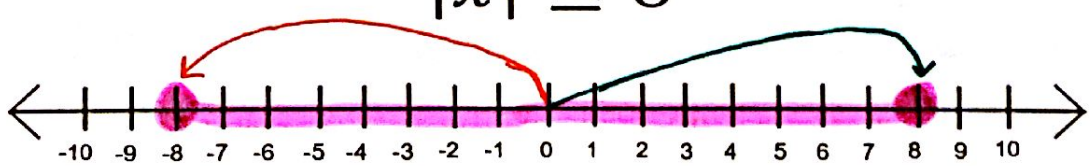


$x<-3$ or $x>3$

- removed abs. value
- flipped the inequality
- made right side opposite

- removed Abs value

$|x| \leq 8$



$x \geq -8$ and $x \leq 8$

- removed abs val.
- flipped neg
- right side opp.

- removed abs val.

- Isolate the absolute value FIRST
- Separate into two inequalities
  - On the left - just remove the absolute value
  - On the right - remove absolute value, flip sign and make right side negative
  - Connect with the word "and" or "or"
    - "and" if $<$ or $\leq$
    - "or" if $>$ or $\geq$
- Solve each separately & graph

Example 4: Solve the absolute value equations.

a. $|4x - 6| \geq 10$

$4x-6 \geq 10$ or $4x-6 \leq -10$

$4x \geq 16$ | $4x \leq -4$

$x \geq 4$ or $x \leq -1$

$(-\infty, -1] \cup [4, \infty)$

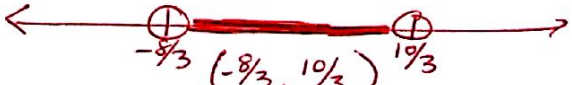
b. $|6y - 2| + 4 < 22$

$|6y-2| < 18$

$6y-2 < 18$ and $6y-2 > -18$

$6y < 20$ | $6y > -16$

$y < \frac{10}{3}$ and $y > \frac{-8}{3}$



$(-8/3, 10/3)$

c. $3|2x - 1| \geq 21$

$|2x-1| \geq 7$

$2x-1 \geq 7$ or $2x-1 \leq -7$

$2x \geq 8$ | $2x \leq -6$

$x \geq 4$ or $x \leq -3$

$(-\infty, 3] \cup [4, \infty)$

d. $2|y - 3| + 5 \geq 29$

$2|y-3| \geq 24$

$|y-3| \geq 12$

$y-3 \geq 12$ or $y-3 \leq -12$

$y \geq 15$ or $y \leq -9$

$(-\infty, -9] \cup [15, \infty)$

e. $|x - 6| + 9 < 3$

$|x-6| < -6$

↑ never < neg #s

$\emptyset$

f. $|6x + 3| > -9$

↑ always > neg #s

$\mathbb{R}$